How Pusheen uses computer to do mathematics

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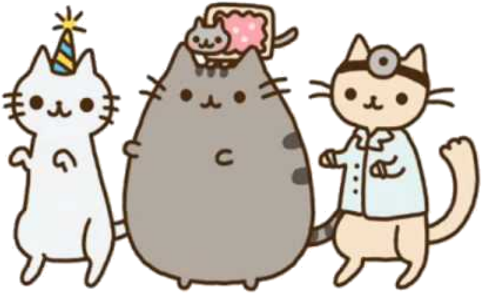

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1. 1089 and 2178, the magic numbers!

2. CoCon 2015

Reverse multiples

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- We observed that if n is a *palindromic number* then

$$\text{rev}_{10}(n) = 1 \times n$$

where $\text{rev}_{10}(n)$ denotes the reverse order number n in the decimal base, i.e.,

$$\text{if } n = (\alpha_N \dots \alpha_1 \alpha_0)_{10}, \ \text{ then } \ \text{rev}_{10}(n) = (\alpha_0 \alpha_1 \dots \alpha_N)_{10}.$$

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  Conjecture (G. J. Simons): "There is no palindrome of the form n^ℓ for $\ell > 4$."
  Conjecture (N. J. A. Sloane?): "If k^4 is a palindrome, then $k = 100\ldots001$."

Formal definition

Let $g \geq 2$ and $1 \leq k < g$. A number n is called a (g, k)-*reverse multiple* iff

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- ... so, we write a program!

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- Lets take a look to another numbers...

- 9-digit:

  219999978

  217802178

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  219999978

  217802178

- 10-digit:

  2199999978

  2178002178

  2197821978

- 9-digit:

  219999978

  217802178

- 10-digit:

  2199999978

  2178002178

  2197821978

- 11-digit:

  21999999978

  21780002178

  21978021978

- 9-digit:

  219999978

  217802178

- 10-digit:

  2199999978

  2178002178

  2197821978

- 11-digit:

  21999999978

  21780002178

  21978021978

- 12-digit:

  219999999978

  217800002178

  219978002199978

  21999978219999978

  217821782178

- 9-digit:

  219999978

  217802178

- 10-digit:

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  2197821978

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  21978021978

- 12-digit:

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- Are you still able to follow the pattern?

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...and similarly for the case of $(10, 9)$-reverse multiples with numbers 1089.

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References:

- A. L. Young: *Trees For k-Reverse Multiples*, Fib. Quart. 30 (1992).
- R. Webster, G. Williams: *On the Trail of Reverse Divisors: 1089 and All that Follow*, Math. Spec. 45 (2013).
- N. J. A. Sloane: *2178 And All That*, Fib. Quart. 52 (2014).

A magic trick

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Contents

1. 1089 and 2178, the magic numbers!
2. **CoCon 2015**

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- Thus, there is an infinite number of solutions: numbers which are factorials of an integer.

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- For example, it can be shown (and it is not very hard) that if $n \in \mathcal{A}$, then $m + k > n + 1$.

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Satan conjecture

$$\mathcal{A} = \{10\}.$$

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Can you do that better?

Computational Contest 2015

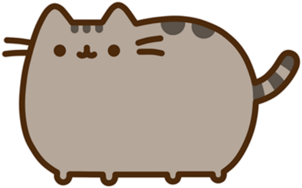
- Can you significantly improve the range of numbers that do (not) belong to $\mathcal{A}$?
- Can you disprove the Satan conjecture? What is the respective formula $x!y! = z!$?
- Apart from the computational properties, can you show something mathematically interesting about $\mathcal{A}$?

The End

Starring:

The End

Starting:



Pusheen

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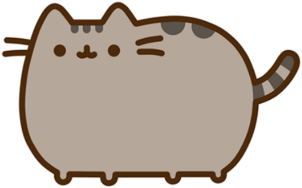

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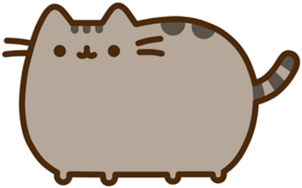

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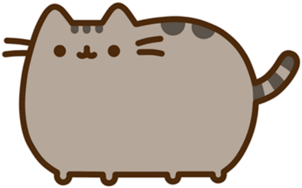

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Thank you for your attention!